Э.В. Сперанская

# A Level Mathematics For Russian pupils

УЧЕБНОЕ ПОСОБИЕ

Москва
ИЗДАТЕЛЬСКАЯ ГРУППА
«ГЭОТАР-Медиа»
2019

**_Пример экзаменационной работы Examination style paper (I)_**

1) Given that $x = 4(3^{-y})$, express y in terms of x. [3]

2) Solve the inequality $2x > |x-1|$. [4]

3) The parametric equations of a curve are $x = 2\theta + \sin 2\theta$, $y = 1 - \cos 2\theta$.

Show that $\frac{dy}{dx} = \tan\theta$. [5]

4) (I) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R >0 and $0° < \alpha < 90°$, giving the exact value of R and the value of $\alpha$ correct to 2 decimal places.

(II) Hence solve the equation

$7\cos\theta + 24\sin\theta = 15$,

giving all solutions in the interval $0° \le \alpha \le 360°$. [4]

5) In a certain industrial process, a substance is being in a container. The mass of the substance in the container t minutes after the start of the process is x grams. At any time, the rate of formation of the substance is proportional to its mass. Also, throughout the process, the substance is removed from the container at a constant rate of 25 gram per minute. When t=0, x=1000 and $\frac{dx}{dt} = 75$

(I) Show that x and t satisfy the differential equation $\frac{dx}{dt} = 0{,}1(x - 250)$. [2]

(II) Solve this differential equation, obtaining an expression for x in terms of t. [6]

6) (I) By sketching a suitable pair of graphs, show that the equation

$2\cot x = 1 + e^x$ where x is in radians, has only one root in the interval $0 < x \le \frac{1}{2}\pi$. [2]

(II) Verify by calculation that this root lies between 0.5 and 1.0. [2]

(III) Show that this root also satisfies the equation $x = \tan^{-1}(\frac{2}{1+e^x})$ [1]

(IV) Use the iterative formula $x_{n+1} = \tan^{-1}(\frac{2}{1+e^{x_n}})$

With initial value $x_1$=0.7, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

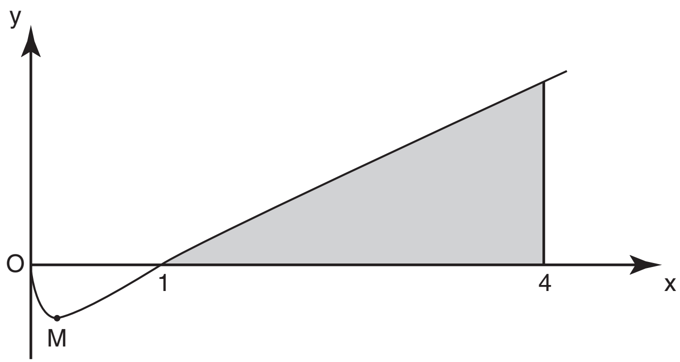
7) The complex number 2+i is denoted by u. Its complex conjugate is denoted by u*.

(I) Show, on a sketch of an Argand diagram with origin O, the points A,B and C representing the complex numbers u, u* and u+u* respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

(II) Express $\frac{u}{u^*}$ in the form x+iy, where x and y are real. [3]

(III) By considering the argument of $\frac{u}{u^*}$, or otherwise, prove that $\tan^{-1}(\frac{4}{3}) = 2\tan^{-1}(\frac{1}{2})$.

8)




The diagram shows a sketch o the curve $y = x^{\frac{1}{2}} \ln x$ and its minimum point M. The curve cuts the x-axis at the point (1.0).

(I) Find the exact value of the x-coordinate of M. [4]

(II) Use integration by parts to find the area of the shaded region enclosed by the curve, the x-axis and the line x=4. Give your answer correct to 2 decimal places. [5]

9) (I) Express $\frac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

(II) Hence, given that $|x|<1$, obtain the expansion of $\frac{10}{(2-x)(1+x^2)}$ in ascending powers of x, up to and including the term in $x^3$, simplifying the coefficients. [5]

10) The points A and B have position vectors, relative to the origin O, given by

$\vec{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$.

The line l passes through A and is parallel to OB. The point N is the foot of the perpendicular from B to l.

(I) State a vector equation from the line l. [1]

(II) Find the position vector of N and show that BN=3. [6]

(III) Find the equation of the plane containing A, B and N, giving your answer in the form ax+by+cz=d. [5]

## Базовый уровень C2 Core Mathematics C2

| Функции | Algebra and functions |
|---|---|

| 1. Деление многочленов | 1. Division of polynomials |
|---|---|
| ***Теория***<br>При делении одного многочлена a(x) на другой b(x) делимое, делитель и частное связаны соотношением $a(x) \equiv b(x)q(x) + r(x)$, при условии, что b(x) не является константой | ***Theory***<br>When a polynomial, a(x), is divided by non – constant divisor, b(x), the quotient q(x) and the remainder r(x) are defined by the identity $a(x) \equiv b(x)q(x) + r(x)$ |
| ***Пример***<br>Найдите частное и остаток при делении $x^4+x+2$ на x+1 | ***Example***<br>Find the quotient and remainder when $x^4+x+2$ is divided by x+1 |
| ***Решение***<br>1) $x^4 + x + 2 \equiv (x+1)(Ax^3 + Bx^2 + Cx + D) + R$<br>2) $x^4 + x + 2 \equiv Ax^4 + (A+B)x^3 + (B+C)x^2 + (C+D)x + D + R$<br>3) 1=A<br>4) 0=A+B, B= −1<br>5) 0=B+C, C=1<br>6) 1=C+D, D=0<br>7) 2=D+R, R=2<br>8) Частное равно $x^3 - x^2 + x$, остаток равен 2 | ***Solution***<br>1) $x^4 + x + 2 \equiv (x+1)(Ax^3 + Bx^2 + Cx + D) + R$<br>2) $x^4 + x + 2 \equiv Ax^4 + (A+B)x^3 + (B+C)x^2 + (C+D)x + D + R$<br>3) 1=A<br>4) 0=A+B, B= -1<br>5) 0=B+C, C=1<br>6) 1=C+D, D=0<br>7) 2=D+R, R=2<br>8) The quotient is $x^3 - x^2 + x$ and remainder is 2 |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Find the quotient and remainder when $x^4 + 3x^2 - 2$ is divided by $x^2 - 2x + 2$ | Find the quotient and remainder when the first polynomial is divided by the second. $x^4 - 2x^3 - 7x^2 + 5$ , $x^2 + 2x - 1$ |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

| 2. Теорема о целых корнях | 2. The factor theorem |
|---|---|
| ***Теория***<br>Дан многочлен p(x)<br>a) Если (x-t) делитель p(x), то p(t) = 0<br>b) Если p(t) = 0, то (x-t) делитель p(x)<br>Если целое число t — корень многочлена p над Z, то для любого целого x: число f (x) кратно (x-t). Это утверждение называется теоремой о целых корнях | ***Theory***<br>Let p(x) be a polynomial. Then<br>a) if (x-t) is a factor of p(x), then p(t) = 0<br>b) if p(t) = 0, then (x-t) is a factor of p(x)<br>The second of this results is called the factor theorem |
| ***Пример***<br>Найдите делители и корни многочлена $x^3 - x^2 - 5x - 3$. Решите уравнение $x^3 - x^2 - 5x - 3 = 0$ | ***Example***<br>Find the factors of $x^3 - x^2 - 5x - 3$, and hence solve the equation $x^3 - x^2 - 5x - 3 = 0$ |
| ***Решение***<br>1) $p(x) = x^3 - x^2 - 5x - 3$<br>2) Пусть x=1,<br>$p(x) = x^3 - x^2 - 5x - 3 = 0\, p(1) = 1^3 - 1^2 - 5\times 1 - 3 = -8 \neq 0$<br>3) Пусть x=−1, $p(-1) = (-1)^3 - (-1)^2 - 5\times(-1) - 3 = 0$ следовательно, (x+1) является делителем<br>4) $x^3 - x^2 - 5x - 3 \equiv (x+1)(x^2 - 2x - 3) = (x+1)^2(x-3)$, x=-1, x=3 | ***Solution***<br>1) $p(x) = x^3 - x^2 - 5x - 3$<br>2)Try x=1,<br>$p(x) = x^3 - x^2 - 5x - 3 = 0\, p(1) = 1^3 - 1^2 - 5\times 1 - 3 = -8 \neq 0$<br>3)Try x=-1, $p(-1) = (-1)^3 - (-1)^2 - 5\times(-1) - 3 = 0$, so (x+1) is a factor<br>4) $x^3 - x^2 - 5x - 3 \equiv (x+1)(x^2 - 2x - 3) = (x+1)^2(x-3)$, x=-1, x=3 |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Show that $(x-1)$ is a factor of $6x^3+11x^2-5x-12$ and find the other two linear factors of the expression | Find the value of ***a*** for which $(x-2)$ is a factor of $3x^3+ax^2+x-2$ |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

| 3. Обобщённая теорема о корне многочлена | 3. The factor theorem (extended form) |
|---|---|
| ***Теория***<br>Дан многочлен p(x).<br>a) Если (sx-t) делитель p(x), то p($\frac{t}{s}$)=0<br>b) Если p($\frac{t}{s}$)=0, (sx-t) делитель p(x)<br>Если число $\frac{t}{s}$ — корень многочлена p над Z, то для любого целого x: число f (x) кратно (sx-t). Это утверждение называется обобщённой теоремой о целых корнях | ***Theory***<br>Let p(x) be a polynomial. Then<br>a) if (sx-t) is a factor of p(x), then p($\frac{t}{s}$)=0<br>b) if p($\frac{t}{s}$)=0, then (sx-t) is a factor of p(x)<br>The second of this results is called ***the extended form*** of the factor theorem |
| ***Пример***<br>Найдите делители и корни многочлена $p(x) \equiv 3x^3 + 4x^2 + 5x - 6$ | ***Example***<br>Find the factors of $p(x) \equiv 3x^3 + 4x^2 + 5x - 6$ |
| ***Решение***<br>1) $s = \pm 1; \pm 3$ .<br>2) $t = \pm 1; \pm 2; \pm 3; \pm 6$ .<br>3) Подставляя эти значения в многочлен, получим: $p(\frac{2}{3}) \equiv 3 \times (\frac{2}{3})^3 + 4 \times (\frac{2}{3})^2 + 5 \times (\frac{2}{3}) = 0$, следовательно, (3x-2) является делителем<br>4) $p(x) \equiv 3x^3 + 4x^2 + 5x - 6 \equiv (3x-2)(x^2 + 2x + 3)$ | ***Solution***<br>1) $s = \pm 1; \pm 3$<br>2) $t = \pm 1; \pm 2; \pm 3; \pm 6$<br>3) Working through all these in turn, find that $p(\frac{2}{3}) \equiv 3 \times (\frac{2}{3})^3 + 4 \times (\frac{2}{3})^2 + 5 \times (\frac{2}{3}) = 0$, so (3x-2) is a factor<br>4) $p(x) \equiv 3x^3 + 4x^2 + 5x - 6 \equiv (3x-2)(x^2 + 2x + 3)$ |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Factorize cubic polynomials p(x).<br>1) $3x^3 - x^2 - 12x + 4$<br>2) $6x^3 + 7x^2 - x - 2$ | Solve the equation<br>$4x^3 + 12x^2 + 5x - 6 = 0$ |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

| 4. Теорема о делении с остатком | 4. The remainder theorem |
|---|---|
| ***Теория***<br>При делении многочлена p(x) на sx-t остаток является постоянной величиной и находится из условия $p(\frac{t}{s})$ | ***Theory***<br>When p(x) is divided by *sx-t*, the remainder is the constant $p(\frac{t}{s})$ |
| ***Пример***<br>Найдите остаток от деления $x^3$-3x+4 на 2x+3. | ***Example***<br>Find the remainder when $x^3$-3x+4 is divided by 2x+3 |
| ***Решение***<br>1) $p(x)=x^3-3x+4$<br>2) Величина остатка: $p(-\frac{3}{2})=(-\frac{3}{2})^3-3\times(-\frac{3}{2})+4=5\frac{1}{8}$ | ***Solution***<br>1) $p(x)=x^3-3x+4$<br>2) $p(-\frac{3}{2})=(-\frac{3}{2})^3-3\times(-\frac{3}{2})+4=5\frac{1}{8}$ |

<table>
<tr><th colspan="2">Work paper</th></tr>
<tr><td>Assignment 1</td><td>Assignment 2</td></tr>
<tr><td>Find the quotient and remainder when the first polynomial is divided by the second.<br>$x^3 + 3x^2 - 2x + 1$<br>2x–1</td><td>Find the quotient and remainder when the first polynomial is divided by the second.<br>$2x^3 + 5x^2 - 3x + 6$<br>3x+1</td></tr>
<tr><td><u>Solution</u></td><td><u>Solution</u></td></tr>
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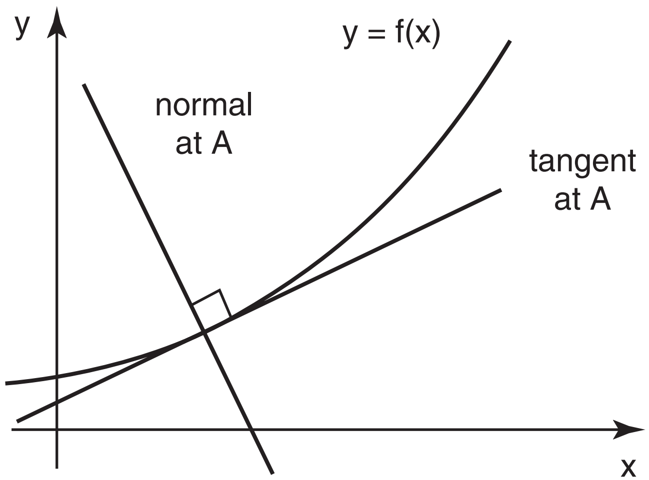
| Геометрия на координатной плоскости | Coordinate geometry in the (x, y) plane |
|---|---|

## 5. Нормаль к кривой в заданной точке

***Теория***

Прямая, проходящая через точку касания касательной к кривой и перпендикулярная к этой прямой, называется нормалью к кривой в заданной точке.

Если тангенс угла наклона касательной к оси Ох равен m, то тангенс угла наклона нормали к оси Ох равен $-\frac{1}{m}$

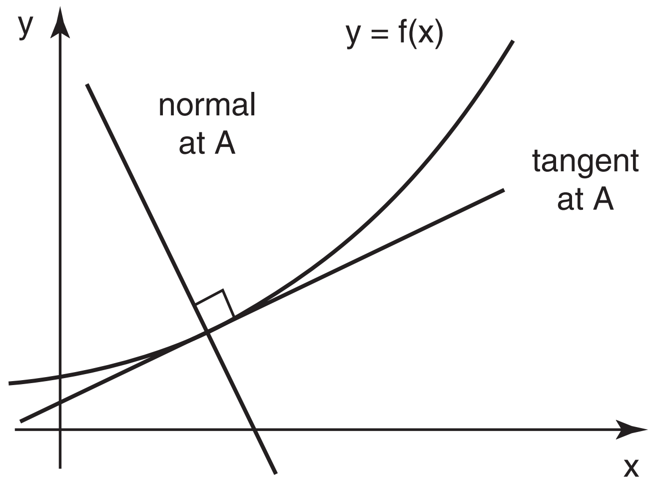



## 5.The normal to a curve at a point

***Theory***

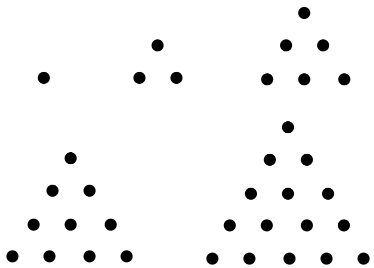
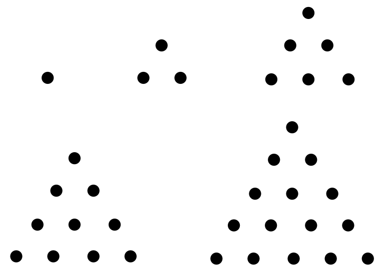
The line passing through the point of contact of the tangent with the curve which is perpendicular to the tangent is called the normal to a curve at a point.

If the gradient of the tangent is m, then the gradient of the normal is $-\frac{1}{m}$



| | |
|---|---|
| ***Пример***<br>Найти уравнение нормали к кривой $y=x^2$ в точке с абсциссой $x = -3$ | ***Example***<br>Find the equation of the normal to the curve $y=x^2$ at the point for which $x = -3$ |
| ***Решение***<br>1) Формула тангенса угла наклона касательной к кривой $y=x^2$ есть 2x. Для нашего случая он составляет −6<br>2) Тангенса угла наклона нормали $-\frac{1}{-6}=\frac{1}{6}$<br>3) Уравнение прямой, проходящей через точку ($x_1$, $y_1$) и имеющей тангенс угла наклона, имеет вид: $y-y_1=m(x-x_1)$<br>4) Уравнение нормали в точке $(-3,9)$ : $y-9=\frac{1}{6}(x-(-3))$ .<br>После упрощения имеем: 6y=x+57 | ***Solution***<br>1) The gradient formula for the curve $y=x^2$ is 2x. So, the gradient is −6<br>2) The normal has a gradient of $-\frac{1}{-6}=\frac{1}{6}$<br>3) The equation of the line through ($x_1$, $y_1$) with gradient m is $y-y_1=m(x-x_1)$<br>4) Therefore the equation of the normal at $(-3,9)$ is $y-9=\frac{1}{6}(x-(-3))$, which simplifies to 6y=x+57 |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Find the gradient of the tangent to the graph of $y=x^2$, at each of the points with the given x-coordinate.<br>a) 1; b) 4; c) 0 | The y- coordinate of a point P on the graph $y=x^2+5$ is 9. Find the two possible values of the gradient of the tangent to $y=x^2+5$ at P |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

| **Бином Ньютона** | **The binomial expansion** |
|---|---|
| ***6. Треугольные числа*** | ***6. The triangle number sequence*** |
| ***<u>Теория</u>***<br>Число кружочков в «треугольных» образцах называют треугольными числами.<br><br>К треугольным числам относятся 1, 3, 4, 10, 15, 21. Разность между двумя последовательными числами увеличивается на 1.<br>Формула для нахождения суммы натуральных чисел от 1 до r: $\frac{1}{2}r(r+1)$ | ***<u>Theory</u>***<br>The numbers of dots in the triangular patterns are called triangle numbers.<br><br>The triangle number are 1, 3, 4, 10, 15, 21. The difference between neighboring triangle numbers increases by 1 each time.<br>The sum of the natural numbers from 1 to r is $\frac{1}{2}r(r+1)$ |
| ***<u>Пример</u>***<br>Десятый образец треугольных чисел имеет 55 кружочков. Сколько будет кружочков в 11 образце? | ***<u>Example</u>***<br>The $10^{th}$ pattern of triangle numbers has 55 dots. How many dots will the $11^{th}$ pattern have? |

| Решение | Solution |
|---|---|
| 1) Сумма натуральных чисел от 1 до r есть $\frac{1}{2}r(r+1)$, где r=11.<br>2) Одиннадцатый образец имеет $\frac{1}{2}11(11+1) = 66$ кружочков | 1) The sum of the natural numbers from 1 to r is $\frac{1}{2}r(r+1)$, r=11<br>2) The 11th pattern has $\frac{1}{2}11(11+1) = 66$ dots |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Find the sum of the natural numbers from 1 to 100 inclusive | Find and simplify an expression for the sum of the natural numbers from (n+1) to 2n inclusive |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

| 7. Последовательность Паскаля | 7. Pascal sequence |
|---|---|
| ***Теория***<br>Последовательность Паскаля в общем виде: $\binom{n}{r+1}=\frac{n-r}{r+1}\binom{n}{r}$, где r = 0, 1, 2, 3 … | ***Theory***<br>The general definition of a Pascal sequence, whose terms are denoted by $\binom{n}{r}$, is $\binom{n}{r+1}=\frac{n-r}{r+1}\binom{n}{r}$, where r = 0, 1, 2, 3 … |
| ***Пример***<br>Составить последовательность Паскаля для n=5 | ***Example***<br>Find the Pascal sequence for n=5 |
| ***Решение***<br>$p_0=1$;<br>$p_1=\frac{5-0}{0+1}\times 1=5$; $p_2=\frac{5-1}{1+1}\times 5=10$; $p_3=\frac{5-2}{2+1}\times 10=10$.<br>$p_4=\frac{5-3}{3+1}\times 10=5$; $p_5=\frac{5-4}{4+1}\times 5=1$; $p_6=\frac{5-5}{5+1}\times 1=0$.<br>Последовательность Паскаля для n=5 имеет вид: 5; 10; 10; 5; 1; 0 | ***Solution***<br>$p_0=1$;<br>$p_1=\frac{5-0}{0+1}\times 1=5$; $p_2=\frac{5-1}{1+1}\times 5=10$; $p_3=\frac{5-2}{2+1}\times 10=10$;<br>$p_4=\frac{5-3}{3+1}\times 10=5$; $p_5=\frac{5-4}{4+1}\times 5=1$; $p_6=\frac{5-5}{5+1}\times 1=0$.<br>The Pascal sequence for n=5 is: 5; 10; 10; 5; 1; 0 |

| ***Work paper*** | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Find the Pascal sequence for n=6 | Use the formula $\begin{pmatrix} n \\ r \end{pmatrix} = \dfrac{n!}{r! \times (n-1)!}$ to write the following in terms of factorials:<br>a) $\begin{pmatrix} 11 \\ 4 \end{pmatrix}$ b) $\begin{pmatrix} 11 \\ 7 \end{pmatrix}$ |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

## 8. Треугольник Паскаля

***Теория***

Приведённая схема, составленная из коэффициентов последовательности Паскаля, называется треугольником Паскаля. В этом треугольнике на вершине и по бокам стоят единицы. Каждое число равно сумме двух расположенных над ним чисел.

**1**

**1 1**

**1 2 1**

**1 3 3 1**

**1 4 6 4 1**

***Пример***

Рассчитать 4-й ряд треугольника Паскаля

***Решение***

1) Последовательность Паскаля для n=4: 5; 10; 10; 5; 1; 0.

2) Треугольник Паскаля:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | | | | | |
| | | | 1 | | 2 | | 1 | | | |
| | | 1 | | 3 | | 3 | | 1 | | |
| | 1 | | 4 | | 6 | | 4 | | 1 | |
| 1 | | 5 | | 10 | | 10 | | 5 | | 1 |

## 8. Pascal's triangle

***Theory***

The complete pattern of Pascal sequence, without the trailing zeros, is called Pascal's triangle. Every number in this pattern except for the first is the sum of the two numbers most closely above it.

**1**

**1 1**

**1 2 1**

**1 3 3 1**

**1 4 6 4 1**

***Example***

Continue Pascal's triangle for row number 4

***Solution***

1) The Pascal sequence for n=4 is: 5; 10; 10; 5; 1; 0

2) Pascal's triangle is

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | | | | | |
| | | | 1 | | 2 | | 1 | | | |
| | | 1 | | 3 | | 3 | | 1 | | |
| | 1 | | 4 | | 6 | | 4 | | 1 | |
| 1 | | 5 | | 10 | | 10 | | 5 | | 1 |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Continue Pascal's triangle for row number 6 | Continue Pascal's triangle for row number 7 |
| ***Solution*** | ***Solution*** |

| 9. Сочетания. Определение факториала | 9. Combinations and factorial notations |
|---|---|
| **Теория**<br>1) $n! = n\times(n-1)\times(n-2)\times(n-3)\times\ldots\times 3\times 2\times 1$<br>2) По определению, $0! = 1$<br>3) Количество способов выбора r элементов из группы n записывается $^{n}c$ или $\binom{n}{r}$ и вычисляется по формуле $\frac{n!}{(n-r)!r!}$ | **Theory**<br>1) $n! = n\times(n-1)\times(n-2)\times(n-3)\times\ldots\times 3\times 2\times 1$<br>2) By definition, $0! = 1$<br>3) The number of ways of choosing r items from a group of n items is written $^{n}c$, or $\binom{n}{r}$ and is calculated by $\frac{n!}{(n-r)!r!}$ |
| **Пример**<br>Найдите значения следующих выражений:<br>a) $^{3}C_{2}$ b) $\frac{10!}{9!}$ | **Example**<br>Find the value of the following:<br>a) $^{3}C_{2}$ b) $\frac{10!}{9!}$ |
| **Решение**<br>a) $^{3}C_{2} = \frac{3!}{(3-2)!2!} = \frac{6}{1\times 2} = 3$<br>b) $\frac{10!}{9!} = \frac{1\times 2\ldots\times 9\times 10}{1\times 2\ldots\times 9} = 10$ | **Solution**<br>a) $^{3}C_{2} = \frac{3!}{(3-2)!2!} = \frac{6}{1\times 2} = 3$<br>b) $\frac{10!}{9!} = \frac{1\times 2\ldots\times 9\times 10}{1\times 2\ldots\times 9} = 10$ |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Calculate:<br>a) $4!$ b) ${}^{10}C_9$ | Calculate:<br>a) $\binom{4}{1}$ b) ${}^{6}C_3$ |
| ***Solution*** | ***Solution*** |

| 10. Последовательность факториалов | 10. The factorial sequence |
|---|---|
| **Теория**<br>1) Факториал числа n (обозначается n!) — это произведение всех натуральных чисел от 1 до n включительно.<br>Запомните 0!=1; 1!=1 r!=1×2×3×4…r<br>2) Формула для расчёта последовательности факториалов $f_{r+1} = f_r \times (r+1)$, где r = 0, 1, 2, 3…<br>$f_0 = 1$<br>$f_1 = f_0 \times 1 = 1 \times 1 = 1$<br>$f_2 = f_1 \times 2 = 1 \times 2 = 2$<br>$f_3 = f_2 \times 2 = 2 \times 3 = 6$<br>Последовательность факториалов неотрицательных целых чисел начинается так:<br>1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800, 479001600, 6227020800.<br>3) Формулы Стирлинга для расчёта приближённого значения произведения n первых натуральных чисел<br>$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ | **Theory**<br>1) The factorial of a non-negative integer n, denoted by n!, is the product of all positive integers less than or equal to n.<br>Remember: 0!=1; 1!=1 r!=1×2×3×4…r<br>2) To get the factorial sequence use the formula $f_{r+1} = f_r \times (r+1)$, where r = 0, 1, 2, 3…<br>$f_0 = 1$<br>$f_1 = f_0 \times 1 = 1 \times 1 = 1$<br>$f_2 = f_1 \times 2 = 1 \times 2 = 2$<br>$f_3 = f_2 \times 2 = 2 \times 3 = 6$<br>The first terms of the factorial sequence are:<br>1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800, 479001600, 6227020800,<br>3) Stirling's approximation (or Stirling's formula) is an approximation for large factorials.<br>$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ |

| | |
|---|---|
| ***<u>Пример</u>***<br>a) Упростите выражение $\frac{14!}{13!}$<br>b) Вычислите 5!, используя формулу Стирлинга | ***<u>Example</u>***<br>a) Simplify the following $\frac{14!}{13!}$<br>b) Calculate 5!, using Stirling's formula |
| ***<u>Решение</u>***<br>a) $\frac{14!}{13!}=\frac{1\times 2\times 3\ldots 14}{1\times 2\times 3\ldots 13}=14$<br>b) $5!\approx\sqrt{2\cdot 3{,}14\cdot 5}(\frac{5}{2{,}71})^5\approx 199{,}999$ | ***<u>Solution</u>***<br>a) $\frac{14!}{13!}=\frac{1\times 2\times 3\ldots 14}{1\times 2\times 3\ldots 13}=14$<br>b) $5!\approx\sqrt{2\cdot 3.14\cdot 5}(\frac{5}{2.71})^5\approx 199.999$ |

| Work paper | |
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| ***Assignment 1*** | ***Assignment 2*** |
| Write the following in terms of factorials:<br>a) $n(n-1)(n-2)$ b) $(n+6)(n+5)(n+4)$ | Calculate 6!, using Stirling's formula |
| ***<u>Solution</u>*** | ***<u>Solution</u>*** |

| 11. Разложение $(x+y)^n$, n >0 | 11. Expansion of $(x+y)^n$, n >0 |
|---|---|
| ***Теория***<br>Для натурального n бином Ньютона имеет вид:<br>$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$ | ***Theory***<br>The binomial theorem states that, if number is a natural number $(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$ |
| ***Пример***<br>Найдите первые три члена в разложении $(1+x)^{22}$ | ***Example***<br>Find the first three terms in the expansion in ascending power of x of the following: $(1+x)^{22}$ |
| ***Решение***<br>1) $\binom{22}{0} = 1$<br>2) $\binom{22}{1}x = \frac{22!}{1!(22-1)!}x = 22x$<br>3) $\binom{22}{2}x^2 = \frac{22!}{2!(22-2)!}x^2 = 231x^2$ | ***Solution***<br>1) $\binom{22}{0} = 1$<br>2) $\binom{22}{1}x = \frac{22!}{1!(22-1)!}x = 22x$<br>3) $\binom{22}{2}x^2 = \frac{22!}{2!(22-2)!}x^2 = 231x^2$ |

| ***Work paper*** | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Find the first three terms in the expansion, in ascending power of x, of $(1+2x)^8$. By substituting x = 0.01, find an approximation to $1.02^8$ | Find the first three terms in the expansion, in ascending power of x, of $(2+5x)^{12}$. By substituting a suitable value for x, find an approximation to $2.005^{12}$ to 2 decimal places |
| ***Solution*** | ***Solution*** |

| ***12. Разложение (a + bx)$^n$, n >0*** | ***12. Expansion of (a + bx)$^n$, n >0*** |
|---|---|
| ***<u>Теория</u>***<br>1)<br>$(a+bx)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}bx + \binom{n}{2}a^{n-2}b^2x^2 + \binom{n}{3}a^{n-3}b^3x^3 \ldots + \binom{n}{n}b^nx^n$<br>2) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ | ***<u>Theory</u>***<br>1)<br>$(a+bx)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}bx + \binom{n}{2}a^{n-2}b^2x^2 + \binom{n}{3}a^{n-3}b^3x^3 \ldots + \binom{n}{n}b^nx^n$<br>2) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ |
| ***<u>Пример</u>***<br>Найдите первые четыре члена разложения (2-3x)$^{10}$ | ***<u>Example</u>***<br>Find the first four terms in the expansion of (2-3x)$^{10}$ |
| ***<u>Решение</u>***<br>$(2-3x)^{10} = 2^{10} + \binom{10}{1}\times 2^9 \times (-3x) + \binom{10}{2}\times 2^8 \times (-3x)^2$<br>$+\binom{10}{3}\times 2^7 \times (-3x)^3 \ldots =$<br>$= 1024 - 10\times 512\times 3x + \frac{10\times 9}{1\times 2}\times 256\times 9x^2 -$<br>$\frac{10\times 9\times 8}{1\times 2\times 3}\times 128\times 27x^3 + \ldots =$<br>$= 1024 - 15360x + 103680x^2 - 414720x^3 + \ldots$ | ***<u>Solution</u>***<br>$(2-3x)^{10} = 2^{10} + \binom{10}{1}\times 2^9 \times (-3x) + \binom{10}{2}\times 2^8 \times (-3x)^2$<br>$+\binom{10}{3}\times 2^7 \times (-3x)^3 \ldots =$<br>$= 1024 - 10\times 512\times 3x + \frac{10\times 9}{1\times 2}\times 256\times 9x^2 -$<br>$\frac{10\times 9\times 8}{1\times 2\times 3}\times 128\times 27x^3 + \ldots =$<br>$= 1024 - 15360x + 103680x^2 - 414720x^3 + \ldots$ |

| Work paper | |
|---|---|
| ***Assignment 1*** | ***Assignment 2*** |
| Find the first three terms in the expansion in ascending power of x of $(1-x)^{30}$ | Find the first three terms in the expansion in ascending power of x of $(1-4x)^{18}$ |
| ***Solution*** | ***Solution*** |